Analysis of losses in non-ideal passive components in the Class-E power amplifier

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Abstract— **This paper analyzes some of the losses that occur in non-ideal passive components employed in the Class-E power amplifier. A complete analysis of the Class-E operation in the presence of the switch on-resistance and shunt capacitor ESR has been performed and an analytic dependence of the output efficiency on the parasitic resistances has been derived. New design equations for modified circuit elements are proposed. An overview of other sources of losses in the Class-E circuit is given and the dominant ones are discussed.**

Keywords— **Power amplifier, class E operation, efficiency, passives, parasitics**

I. Introduction

Class-E power amplifiers were first proposed by the two Sokals in 1975. [1]. They belong to the switching type power amplifiers (PA), thus offering higher theoretical efficiency than conventional PA configurations. In addition to that, the Class-E PA brought a unique feature into the family of switching type amplifiers: the so-called soft switching, which enabled spectacular efficiencies to be achieved, infeasible with other topologies. This intriguing circuit has aroused a lot of interest in the scientific circles, and many detailed in-depth studies have been published, particularly in the late seventies and during the eighties. Some of the classic early days work include [2],[3],[4],[6]. Today, with the rapid development of wireless communication systems worldwide, Class-E PAs are becoming more interesting than ever. Their high efficiency makes them attractive for utilization in various portable radio transmitting devices. Through saving the battery power, higher efficiency is reflected in longer operation time and, possibly, smaller, lighter and more reliable handset. These important issues indicate that there is a strong interest to go further in the research of this unusual, but yet promising type of PAs. This is particularly true if we keep in mind that the semiconductor technology is constantly progressing, offering better, faster and more reliable devices than ever.

The Class-E PA is, like any other circuit, followed by the real world difficulties that spoil the performance and cause various undesired effects. The goal in this paper is to analyze losses in the Class-E circuit which are caused by the presence of the parasitic resistances in the switch and shunt capacitor. These losses may be small in comparison with some other types of losses in the circuit, particularly those associated with the active device, but still are not negligible. Moreover, it is interesting to estimate the theoretical limit for the efficiency as a function of these parasitics.

The paper is organized as follows. In the second section, the analysis of the conventional (ideal) Class-E operation is performed. In the third section, we repeat the analysis for the cases when the circuit includes parasitic resistances; one parasitic effect is analyzed at a time.

II. The Class E power amplifier

In this section we will perform an analysis of the Class-E operation and of losses that occur in some of the non-ideal passive components employed in the circuit. The equivalent circuit diagram of the Class-E PA is shown in figure 1. The circuit consists of an active device (which is shown as a switch), shunted by a capacitor C, an RF choke (RFC), a series LC resonator $L_0 - C_0$, an excessive reactance X and a load resistance R_L. The parasitic switch inductance and electrical series resistance are denoted as L_{sw} and r_{on}, respectively, while r_c represents the electrical series resistance of the shunt capacitor. In order to simplify the analysis, it is necessary to make certain assumptions. These assumptions are the following :

1. The transistor is modeled as an ideal switch, i.e. a short circuit in the ON state and an open circuit in the OFF state, with instant switching action.

2. The switch is operated with the 50 % duty-cycle, at the carrier frequency.
3. The switch can sustain the current running through it in the ON state and also must be able to stand the non-zero voltage that appears during the OFF state.
4. The RF choke (DC-feeder) has a very large inductance and accordingly allows only DC current to flow through it.
5. The Q-factor of the series resonator $L_0 - C_0$ is high enough, so it can be considered that a purely sinusoidal current is running through the load R_L.

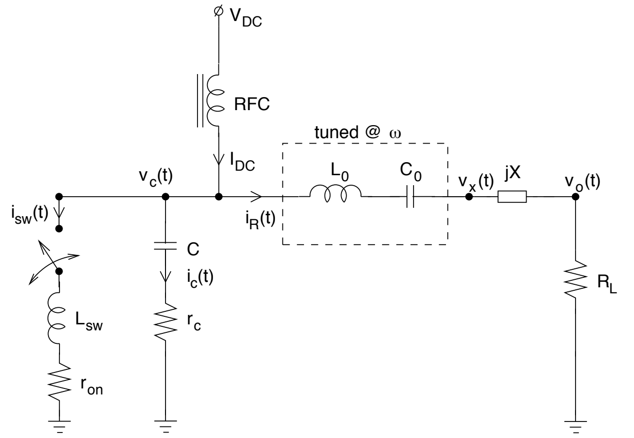

Fig. 1. Equivalent Class-E circuit for analysis

First, a full analysis of the Class-E operation under ideal conditions will be performed, in order to obtain nominal expressions for the circuit elements. Therefore, at first instant we will treat the elements L_{sw}, r_{on} and r_c as non-existent. The analysis proceeds as follows.
We will consider that the circuit has reached a steady-state operation, and will divide an RF cycle in the ON and OFF state. For further analysis, it is convenient to adopt the angular time, given by

$$\theta = \omega t \tag{1}$$

where ω represents the frequency of operation. The ON and OFF states will be defined in time as

$$0 < \theta < \pi \qquad \text{ON state} \tag{2}$$
$$\pi < \theta < 2\pi \qquad \text{OFF state} \tag{3}$$

Due to the very large reactance of the RF choke at the frequency of operation, we can consider that only DC current I_{DC} flows into the circuit. On the other hand, the assumption of a relatively high loaded Q-factor of the series resonator $L_0 - C_0$ suggests that only current at the fundamental frequency is flowing through the load branch. Therefore, we can write

$$i_R(\theta) = I_1 \sin(\theta + \varphi) \tag{4}$$

where I_1 denotes the amplitude of the load current and φ is the initial phase. In the OFF state, the currents flowing through the switch and through the shunt capacitor will be

$$i_{sw}(\theta) = 0 \tag{5}$$

and

$$i_c(\theta) = I_{DC} - I_1 \sin(\theta + \varphi) \tag{6}$$

The current $i_c(\theta)$ is charging/discharging capacitor C. Since we assume that during the ON state the voltage across switch/shunt capacitor is equal to zero, the capacitor voltage in the OFF state can be found as

$$v_c(\theta) = \frac{1}{\omega C} \int_0^\theta i_c(\theta)\, d\theta \tag{7}$$

If we substitute (6) into (7), and perform integration in the given boundaries, we will obtain that the capacitor voltage at any instant in the OFF state is given by

$$v_c(\theta) = \frac{1}{\omega C} \left[I_{DC}\theta + I_1 \cos(\theta + \varphi) - I_1 \cos\varphi \right] \tag{8}$$

A unique feature of the Class-E operation, that distinguishes this class from the other switching-mode PA configurations, is the so-called "soft switching". It means that the circuit operation is arranged in such manner that the shunt capacitor is never discharged through the switch. In other words, the switch closes precisely at the instant where the shunt capacitor is completely discharged. This type of operation enables high efficiency power amplification. Therefore, in order to achieve soft switching, it is necessary to satisfy the well known Class-E conditions at the instant of turn-on (i.e. $\theta = \pi$). These conditions are given as

$$v_c(\theta = \pi) = 0 \tag{9}$$
$$\frac{dv_c(\theta)}{d\theta}\Big|_{\theta=\pi} = 0 \tag{10}$$

It should be mentioned that the second condition (10) actually doesn't refer to the state of the capacitor charge. It describes the slope of the capacitor voltage at the instant of turn-on and guarantees that the switching will not produce current spikes in the switch.
From (8) and (10), we will have

$$I_{DC} = \frac{2}{\pi} \cos\varphi \, I_1 \tag{11}$$

and from (8) and (10)

$$I_{DC} = -\sin\varphi \, I_1 \tag{12}$$

By combining the previous two equations, we find

$$\varphi = \arctan\left(-\frac{2}{\pi}\right) \tag{13}$$

Noticing that I_{DC} and I_1 are both positive values, the solution for the angle φ must be chosen in the right quadrant, i.e. such as to satisfy (11) and (12). Therefore, we obtain the fourth quadrant solution, $\varphi = -0.567\text{rad}$ and

$$\sin\varphi = -\frac{2}{\sqrt{\pi^2+4}} \tag{14}$$

and

$$\cos\varphi = \frac{\pi}{\sqrt{\pi^2+4}} \tag{15}$$

For voltage v_x (fig. 1), we can write

$$v_x(\theta) = I_1\sqrt{R_L^2+X^2}\sin(\theta+\varphi+\psi) \tag{16}$$

where X represents the value of the excessive reactance placed between the series resonator and the load resistance, and ψ is the angle of the impedance $R_L + \mathrm{j}X$, given by

$$\tan\psi = \frac{X}{R_L} \tag{17}$$

By expanding the sine term in (16), we can transform this equation into

$$v_x(\theta) = V_{CI}\sin(\theta+\varphi) + V_{CQ}\cos(\theta+\varphi) \tag{18}$$

where V_{CI} and V_{CQ} are the in-phase and quadrature component of the voltage $v_x(\theta)$, given by

$$V_{CI} = I_1\sqrt{R_L^2+X^2}\cos\psi \tag{19}$$

and

$$V_{CQ} = I_1\sqrt{R_L^2+X^2}\sin\psi \tag{20}$$

Therefore, we can relate the angle ψ with V_{CI} and V_{CQ} as

$$\tan\psi = \frac{X}{R} = \frac{V_{CQ}}{V_{CI}} \tag{21}$$

We will proceed with the analysis to find the components V_{CI} and V_{CQ}.

The series resonator $L_0 - C_0$ represents a short circuit for the fundamental component of current flowing through this branch. Therefore, the two components V_{CI} and V_{CQ} can be determined by finding the Fourier components of the voltage $v_c(\theta)$ with respect to the phase of the current $i_R(\theta)$,

$$V_{CI} = \frac{1}{\pi}\int_0^{2\pi} v_c(\theta)\sin(\theta+\varphi)\,d\theta \tag{22}$$

and

$$V_{CQ} = \frac{1}{\pi}\int_0^{2\pi} v_c(\theta)\cos(\theta+\varphi)\,d\theta \tag{23}$$

Since $v_c(\theta) = 0$ during the ON state ($\pi < \theta < 2\pi$), the upper boundary in the integrals above can be taken to be π. By substituting (8) in (22) and (23), and performing the integration, we will obtain

$$V_{CI} = \frac{1}{\pi\omega C}\left[I_{DC}(\pi\cos\varphi - 2\sin\varphi) - 2I_1\cos^2\varphi\right] \tag{24}$$

$$V_{CQ} = \frac{1}{\pi\omega C}\Big[I_{DC}(-\pi\sin\varphi - 2\cos\varphi) + I_1\frac{\pi}{2} + I_1 2\sin\varphi\cos\varphi\Big] \tag{25}$$

It should be noted that evaluation of the integrals in (22) and (23) will involve terms of the type $\int x\sin x dx$, which should be solved by partial integration.

By substituting (11), (14) and (15) into (24) and (25), we obtain

$$V_{CI} = \frac{I_1}{\omega C}\frac{8}{\pi(\pi^2+4)} \tag{26}$$

and

$$V_{CQ} = \frac{I_1}{\omega C}\frac{\pi(\pi^2-4)}{2(\pi^2+4)} \tag{27}$$

From (21), (26) and 27), it is now easy to find

$$\tan\psi = \frac{\pi(\pi^2-4)}{16} \approx 1.152 \tag{28}$$

Therefore, the value of the excessive reactance equals

$$X = 1.152R_L \tag{29}$$

Furthermore, we can derive the relationship between R_L and C by combining (17), (19) and (26), in order to obtain

$$R_L = \frac{1}{\omega C}\frac{8}{\pi(\pi^2+4)} \tag{30}$$

Therefore, the nominal value of the shunt capacitance equals

$$C = \frac{1}{\omega R_L}\frac{8}{\pi(\pi^2+4)} \approx \frac{0.1836}{\omega R_L} \tag{31}$$

The only derivation left is to relate these parameters to the supply voltage V_{DC}, for the desired level of output power. To do so, we will evaluate the average voltage of $v_c(\theta)$, noticing that it must be equal to the supply voltage. Therefore, we can write

$$\frac{1}{2\pi}\int_0^{2\pi} v_c(\theta)\,d\theta = V_{DC} \tag{32}$$

Again, the integration is performed only in the OFF state, since in the ON state $v_c(\theta) = 0$. By combining (8), (12), (14) and evaluating the integral in (32), we obtain

$$I_{DC}R_L\frac{\pi^2+4}{8} = V_{DC} \tag{33}$$

Thus, the DC resistance seen by the supply source is

$$R_{DC} = R_L \frac{\pi^2 + 4}{8} \approx 1.734 R_L \tag{34}$$

The power delivered to the circuit by the DC power supply is then

$$P_{DC} = V_{DC} I_{DC} = \frac{V_{DC}^2}{R_{DC}} = \frac{V_{DC}^2}{R_L} \frac{8}{\pi^2 + 4} \approx 0.5768 \frac{V_{DC}^2}{R_L} \tag{35}$$

The output RF power delivered to the load R_L can be found as

$$P_{out} = \frac{1}{2} R_L I_1^2 = \frac{1}{2} R_L \left(\frac{I_{DC}}{-\sin\varphi} \right)^2 = \frac{V_{DC}^2}{R_L} \frac{8}{\pi^2 + 4} \tag{36}$$

which is essentially an algebraic confirmation that the DC-to-RF power conversion efficiency ($\eta = P_{out}/P_{DC}$) is equal to 100 %. From equation (36), the required load resistance for the given P_{out} and V_{DC} can be calculated.

In this section we have completed the analysis of an idealized Class-E operation, derived design equations for the nominal element values and confirmed that the theoretical output efficiency equals 100 %. In the next section we will analyze operation of the circuit when some parasitic elements are present.

III. CLASS E PA WITH PARASITIC ELEMENTS

A. *Influence of the ESR of the shunt capacitor*

First we will perform an analysis for the case when the shunt capacitor in the Class-E circuit is not ideal, i.e. we will assume that it has a certain parasitic electrical series resistance (ESR), which is denoted as r_c in figure 1. The other two parasitic elements (L_{sw},r_{on}) will be considered non-existent. The principle of the analysis is the same as in the ideal case, with necessary modifications of some equations. All the assumptions made at the beginning of the previous section hold here as well. The definition of the ON and OFF states are the same as in (2)and (3).

In the OFF state, the shunt capacitor voltage will now be

$$v_c(\theta) = \frac{1}{\omega C} \int_0^\theta i_c(\theta)\, d\theta + r_c i_c(\theta) \tag{37}$$

Thus, by substituting (6) into (37) and evaluating the integral, we will have

$$v_c(\theta) = \frac{1}{\omega C} \left[I_{DC}\theta + I_1 \cos(\theta + \varphi) - I_1 \cos\varphi \right] + r_c \left[I_{DC} - I_1 \sin(\theta + \varphi) \right] \tag{38}$$

The Class-E conditions will also be applied here, with a small modification: the second condition, described by (10), will now refer to the shunt capacitor voltage, and not to the voltage $v_c(\theta)$ (which now represents voltage across the series C-r_c connection). In other words, at the instant of turn-on, the capacitor is fully discharged and the instantaneous current $i_c(\theta)$ is zero. These two conditions result in the same relations between angle ϕ and currents I_{DC} and I_1 as it is the case with the ideal Class-E operation. Therefore, equations (11)-(15) hold.

Also, the presence of r_c does not conflict with the considerations in (16)-(21), so these equation hold as well. Furthermore, equations (22) and (23) are also valid, but for evaluation of the integrals, we have to substitute $v_c(\theta)$ from (38). This results in the following expressions:

$$V_{CI} = \frac{I_1}{\omega C} \left[\frac{8}{\pi(\pi^2+4)} - \omega C r_c \frac{\pi^2 - 4}{2(\pi^2+4)} \right] \tag{39}$$

$$V_{CQ} = \frac{I_1}{\omega C} \left[\frac{\pi^2 - 4}{2(\pi^2+4)} + \omega C r_c \frac{8}{\pi(\pi^2+4)} \right] \tag{40}$$

Therefore, from (21), (39) and (40) we find that

$$\tan\psi = \frac{V_{CQ}}{V_{CI}} = \frac{\pi(\pi^2-4) + 16\omega C r_c}{16 - \pi(\pi^2-4)\omega C r_c} \tag{41}$$

Things are getting more complicated here than in the ideal case in the previous section, because both V_{CI} and V_{CQ} depend on $\omega C r_c$ and $\tan\psi$ can not be directly calculated, as we have done that in (28). In other words, an in advance known value of r_c is not sufficient to enable us to perform an exact calculation of the circuit elements for a given set of input data (P_{out},V_{DC},ω); we need to know $\omega C r_c$. Since r_c is the ESR of the capacitor, the Q-factor of the capacitor at the frequency of operation will be

$$Q_c = \frac{X_c}{r_c} = \frac{1}{\omega C r_c} \tag{42}$$

Thus, from the previous two equations, we can express the dependence of $\tan\psi$ on Q_c as

$$\tan\psi = \frac{\pi(\pi^2-4)Q_c + 16}{16 Q_c - \pi(\pi^2-4)} \tag{43}$$

For an ideal capacitor, Q_c approaches infinity, and (43) reduces into (28). However, a realistic capacitor will have a finite quality factor. For example, discrete SMD capacitors of the order of several pF will have a typical value of the ESR of 0.3Ω, which means that a typical quality factor will be around 30 at $f = 2\text{GHz}$. The dependence of $\tan\psi$ on Q_c, given by (43), is plotted in figure 2.

Needless to say that the relation between R_L and C, given by (30) for the ideal Class-E operation will also be modified. From (17), (19) and (39), we can derive

$$R_L = \frac{1}{\omega C} \left[\frac{8}{\pi(\pi^2+4)} - \frac{1}{Q_c} \frac{\pi^2 - 4}{2(\pi^2+4)} \right] \tag{44}$$

In (2) e (3) sostituire ON con OFF e viceversa.

L'introduzione della X nella (16) si giustifica considerando che Vc (θ) non è sinusoidale , ne trovo quindi la fondamentale con Fourier . La fondamentale di Vc (θ) sarà sfasata con I_R (θ) di un certo angolo ψ , questo giustifica la presenza di una reattanza X.
La (27) presenta un errore di stampa, la formula esatta è :

$$V_{CQ} = \frac{I_1}{\omega C} \frac{(\pi^2 - 4)}{2(\pi^2 + 4)}$$

Il programma calcola i valori dei componenti di un amplificatore in Classe E ideale usando parte della trattazione di Milosevic, Tang, Roermund.
Le formule sono state verificate e corretto l'unico errore di stampa.
Introducendo i dati di progetto vengono calcolati i componenti, vi è pure la possibilità di vedere i grafici di tensioni e correnti in funzione dell'angolo: $\Theta = \omega * t$
E' possibile anche calcolare la rete adattatrice di impedenza per usare carichi standard, ad es: 50 Ω.
La "verifica" calcola la tangente dell'angolo esistente tra la prima armonica della tensione esistente ai capi del condensatore C1 (Vx1) e la corrente Ir (anch'essa ritenuta sinusoidale). Viene calcolata quindi la potenza: Po = ½ Re(Vx1*Ir^)
Ir = Vx1/Z .
Z è l'impedenza complessa che si vede all'ingresso del circuito C2, L2, Rete adattatrice, Carico RL.
Ir è un numero complesso.
Ir^ è il coniugato di Ir.
Con: Re(numero complesso) si intende la parte reale di quel numero complesso.

Per il calcolo di C2 si considera dapprima una capacità Co tale da costituire con L2 un circuito risonante serie:
$1/(\omega Co) = \omega L2$, da cui : $Co = 1/(\omega^2 L2)$, però così Ir sarebbe in fase con Vx1.
Per tener conto dello sfasamento ψ (psi) tra Vx1 e Ir nel circuito viene inserita una reattanza: X = 1,152*R
Per cui sarà: $1/(\omega C2) = 1/(\omega Co) - X$ da cui: $C2 = Co/(1 - 1{,}152*\omega*Co*R)$
La verifica appura che lo sfasamento tra Vx1 e Ir sia appunto ψ (psi) e che la potenza dissipata sul carico sia quella di progetto. Ambedue le condizioni sono soddisfatte.